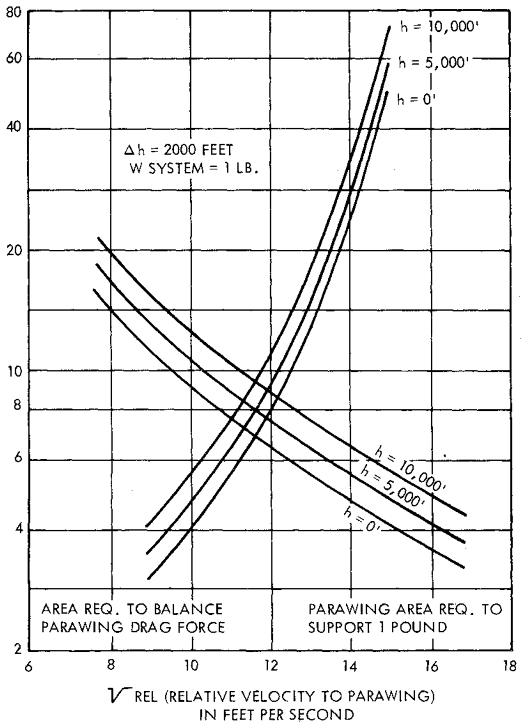

Fig. 2 Required parawing and drag areas.

The parawing characteristics are typical of rigid frame conical canopy parawings,[3,4] conical canopy parawings with large diameter leading edges[5] and the best of the all-flexible twin-keel parawings.[6] Substantially higher L/D ratios can be obtained but they require the use of solid or cylindrical canopies[7] with rigid frames.

Reasonable size parawing and drag areas result if the relative velocity is chosen near 12 fps. If a design point of h = 5000 ft is chosen, the area of the parawing and drag area are A 8.00 ft^2 for V_{REL} = 11.65 fps.

The parawing and drag areas for the design point ($A_p = A_A$ = 8.00 ft^2) have a combined weight (including suspension lines) of two ounces if made from ½ MIL reinforced mylar film. The music wire member which connects the parawing and drag areas is 5800 ft long for Δh = 2000 ft, has a diameter of 0.002 of an inch and weighs one ounce. The weight estimate for the wire assumes a minimum tensile strength of 475,000 psi (Ref. 8) and a factor of safety of 1.5. The system may be packaged in a volume of only a few cubic inches. The system payload may be increased further by the use of a second parawing instead of the drag chute.

The operating altitude of the wind shear system is sensitive to changes in the magnitude of the wind shear since the lift force varies as the square of this quantity. The wind shear system will ascend or descend until a sufficient change in ambient density occurs to produce the original lift force at the new operating altitude and altered wind shear magnitude. For long duration flights where the wind shear may change from the value at launch, flight at the minimum expected wind shear level may be assured by off-loading payload from a given design or by using a system which is sized for this condition. The system may be set up in its flying configuration and ground launched like a kite or dropped from an aircraft.

Conclusions

A wind shear payload support system using two connected aerodynamic bodies which operate at different altitudes and extract energy from wind shear to maintain the system in flight is a feasible concept. An example of such a system which has a parawing for the upper body and a drag area for the lower one is considered for support of a total system weight of one pound. The combined parawing and drag areas are found to total less than 20 ft^2 for an altitute separation of 2000 ft and payloads of 50% or more of the total system weight are possible.

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Vibrations of an Euler Beam with a System of Discrete Masses, Springs, and Dashpots

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Introduction

THE present work analyses the vibrations of an Euler beam carrying masses and having spring-cum-dashpot supports at discrete points. The work is an extension of the work done by Pan[1,2] and generalizes the works done by McBride[3] and Das.[4] The discrete elements are included in the equation of motion through Dirac delta functions. The equations of motion resulting from the eigen expansion are uncoupled with the help of coordinate defined by Foss,[5] where displacements and velocities are treated separately. As an illustration, response to landing impact of an aircraft wing-fuselage-landing gear system is worked out.

Equations of Motion

Consider an Euler beam (length L, mass/unit length m, and rigidity EI) carrying masses M_i, springs K_i, and/or dashpots C_i at distances x_i (i = 1-N). The equation of motion for free vibration $y(x,t)$ may be written as

$$\frac{\partial^4\eta}{\partial\xi^4} + \sum_i^N \psi_i\delta(\xi-\xi_i)\eta + [1+\sum_i^N \mu_i\delta(\xi-\xi_i)]\frac{\partial^2\eta}{\partial\theta^2} + \sum_i^N \zeta_i\delta(\xi-\xi_i)\frac{\partial\eta}{\partial\theta} = 0 \quad (1)$$

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where

$$\xi = x/L; \quad \xi_i = x_i/L; \quad \eta = y/L; \quad \psi_i = K_i L^3/EI$$

$$\mu_i = M_i/mL; \quad \zeta_i = C_i/(mEI)^{1/2}; \quad \theta = t(EI/mL^4)^{1/2}$$

By assuming the solution of Eq. 1 as

$$\eta(\xi,\theta) = \phi(\xi)\exp(-\nu + j\omega)\theta = \phi(\xi)\exp^{(j\lambda^2\theta)} \tag{2}$$

the modes $\phi(\xi)$ are obtained as

$$\phi(\xi) = \frac{\phi(0)}{2}P(\lambda\xi) + \frac{\phi'(0)}{2\lambda}R(\lambda\xi) - \frac{\phi''(0)}{2\lambda^2}Q(\lambda\xi) - \frac{\phi'''(0)}{2\lambda^3}S(\lambda\xi) + \frac{1}{2\lambda^3}\left[\sum_i^N(\psi_i + j\lambda^2\zeta_i - \lambda^4\mu_i)\cdot S(\lambda(\xi-\xi_i))\cdot u(\xi-\xi_i)\right] \tag{3}$$

where $P(z)$, $Q(z) = \cos(z) \pm \cosh(z)$: $R(z)$, $S(z) = \sin(z) \pm \sinh(z)$, z being complex and u is the unit step function.

After eliminating the two known boundary conditions at $\xi = 0$, $(N + 2)$ linear homogeneous equations are written for the unknowns of Eq. (3)with the help of two boundary conditions at $\xi = 1$, and N consistency conditions at $\xi = \xi_i$. The determinant of coefficients then yield the complex frequencies λ_n and the complex modes ϕ_n.

Quasi-Orthogonality Relations

Let $g(\xi,\theta)$ represent the velocity satisfying

$$g(\xi,\theta) = \Gamma(\xi)\cdot e^{j\lambda^2\theta} \tag{4}$$

and

$$j\lambda_\eta^2\phi_\eta - \Gamma_\eta = 0 \tag{5}$$

From Eqs. (1, 2, 4, and 5), by writing $\partial^2\eta/\partial\theta^2$ as $\partial g/\partial\theta$, the following quasi-orthogonality relations are obtained after some algebric manipulations as $d^4/d\xi 4$ is a self-adjoint operator

$$\int_0^1 M(\xi)(\Gamma_\eta\phi_m + \phi_\eta\Gamma_m)d\xi + \sum_i^N\zeta_i\phi_m(\xi_i)\phi_\eta(\xi_i) = 0 \quad m \neq n \tag{6}$$

and

$$\int_0^1[\phi_\eta''\phi_m'' - M(\xi)\phi_\eta\phi_m]d\xi - \sum_i^N\psi_i\phi_\eta(\xi_i)\phi_m(\xi_i) = 0 \quad m \neq n \tag{7}$$

where

$$M(\xi) = 1 + \sum_i^N\mu_i\delta(\xi - \xi_i)$$

Response to Initial Disturbance or External Forces

Modal expansion may be written as

$$\eta(\xi,\theta) = \sum_\eta^\infty\phi_\eta(\xi)T_\eta(\theta) \tag{8}$$

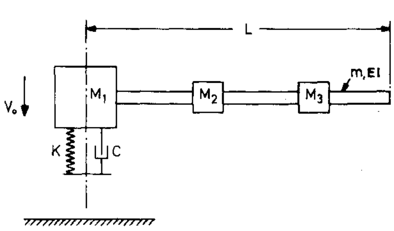

Fig. 1 Aircraft wing - landing gear-fuselage system.

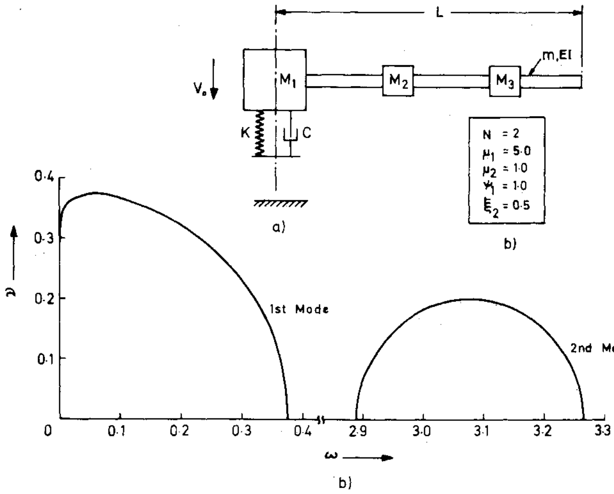

Fig. 2 Complex frequencies.

$$g(\xi,\theta) = \sum_\eta^\infty\Gamma_\eta(\xi)\cdot T_\eta(\theta) \tag{9}$$

The preceding in conjunction with Eqs. (1, 6, and 7) yield uncoupled equations in $Tn'S$ as

$$\dot{T}_\eta(\theta) - j\lambda_\eta^2T_\eta(\theta) = 0 \tag{10}$$

or

$$T_\eta(\theta) = A_\eta\cdot\exp(j\lambda_\eta^2\theta)$$

Equations (6) and (7) are again used to evaluate A_n, from the initial conditions $\eta_0(\xi)$ and $g_0(\xi)$, (displacement and velocity, respectively) as

$$A_\eta = \frac{1}{Q_\eta}\left[\int_0^1 M(\xi)\{\eta_0(\xi)\Gamma_\eta(\xi) + g_0(\xi)\phi_\eta(\xi)\}d\xi + \sum_i^N\zeta_i\phi_\eta(\xi_i)\eta_0(\xi_i)\right] \tag{12}$$

where Q_n is the left-hand side of Eq. (6) evaluated for $m = n$. The response in terms of real quantities is then obtained from Eqs. (2, 8, 11, and 12), by considering frequencies and modes in conjugate pairs, as

$$\eta(\xi,\theta) = \sum_\eta^\infty 2e^{-\nu_\eta\theta}[R_\eta(\xi)\cos(\omega_\eta\theta) + I_\eta(\xi)\sin(\omega_\eta\theta)] \tag{13}$$

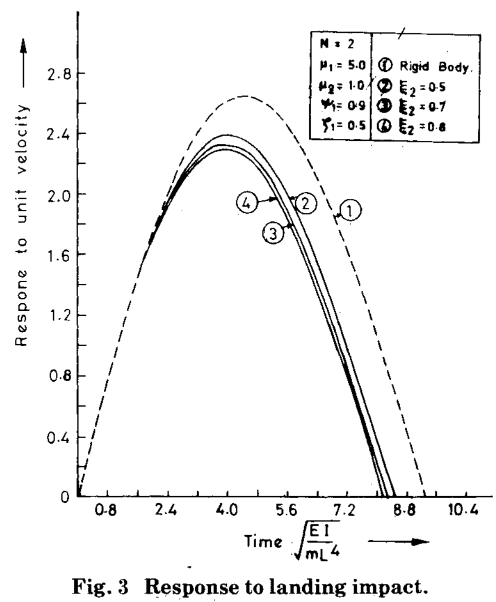

Fig. 3 Response to landing impact.

where

$$A_\eta \phi_\eta(\xi) = R_\eta(\xi) + jI_\eta(\xi)$$

If the beam is subjected to an external force of the form $G_1(\xi)\cdot G_2(\theta)$, the response is then obtained as

$$\eta(\xi,\theta) = 2\sum_{\eta}^{\infty} R_\eta'(\xi)\int_0^\theta \exp -\nu_\eta(\theta-\tau)\cdot\cos(\omega_\eta(\theta-\tau))\cdot G_2(\tau)d\tau - I_\eta'(\xi)\int_0^\theta e^{-\nu_\eta(\theta-\tau)}\cdot\sin(\omega_\eta(\theta-\tau))\cdot G_2(\tau)d\tau \quad (14)$$

where

$$\frac{\phi_\eta(\xi)u_\eta}{Q_\eta} = R_\eta'(\xi) + jI_\eta'(\xi);\quad u_\eta = \int_0^1 G_1(\xi)\phi_\eta(\xi)d\xi$$

Example

An aircraft for the analysis of landing impact is idealized as in Fig. 1, where the wing is considered to be a beam, M_1 is the fuselage mass, M_2 and M_3 are the engine masses and K and C are landing gear stiffness and damping. This is a more realistic model as compared to the one treated by Stowell et al.[6] While writing Eq. (1) for this system ξ_1 is taken equal to $(0 + \epsilon)$.[1,2] The boundary conditions are $\phi'(0) = \phi'''(0) = \phi''(1) = \phi'''(1) = 0$, and the initial conditions are $\eta_0(\xi) = 0$, $g_0(\xi) = V_0$, the dimensionless velocity of descent.

The frequency equation was solved by Newton-Raphson method, and the frequencies for a particular case are shown in Fig. 2. Fuselage motion following landing are plotted in Fig. 3 for different positions of the engine mass. The relation of this response with that of the corresponding ragid body system is a measure of the effect of interaction of wing flexibility with the landing gear forces.

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Errata

Analytical Method for Combining the Interaction of Inlet Distortion and Turbulence

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FIGURE 3 displaying dynamic airfoil data from the experiments of Carta is incorrectly labeled. The upper curve is for high frequency $\omega c/V = 0.60$ and the lower curve is for low frequency $\omega c/V = 0.15$. The author is grateful to one of the reviewers for pointing out this error before publication and to F. O. Carta for pointing it out after publication.

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Graded Thermal Barrier—A New Approach for Turbine Engine Cooling

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IN Table 1, Median Test Results, of the original paper, the entry for graded coating-0.030 Ni-Cr-Mo/ZrO_2 should have a 100+ in the column for Burner Liner Thermal Shock. This specimen did not fail in 100 shocks.

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Index categories: Structural Composite Materials (Including Coatings); Thermal Modeling and Experimental Thermal Simulation.